

# Tensor Decomposition Based Hybrid Wideband Transceiver to Maximize the Achievable Rate in Single-User Multi-Relay MIMO Systems

<sup>1,2</sup>Zunira Abbasi, <sup>1,2</sup>Seung-Geun Yoo, <sup>1,2</sup>Min-A Kim, <sup>1,2</sup>Hafiz Mustafa, <sup>1,2</sup>Hyoung-kyu Song\*

<sup>1</sup> Department of Information and Communication Engineering and <sup>2</sup>Convergence Engineering for Intelligent Drone

Sejong University, Seoul, 209 Neungdong-ro, 05006, Korea

zunira@sju.ac.kr, dbtmdrms96@naver.com, happy990927@naver.com, mustafasurrey@gmail.com, songhk@sejong.ac.kr\*

## Abstract

Hybrid precoding in a single-user multi-relay multi-input multi-output (MIMO) system is a challenging task under wideband assumption, as the element-wise constant amplitude constraint is essentially required for the analog beamforming matrix at each communicating node. To address this problem, multi-linear singular value decomposition (SVD) is exploited to derive the analog beamforming design and then, digital baseband processing component at each frequency carrier can be obtained by diagonalizing the equivalent baseband channels. Simulation result reveals that the proposed algorithm achieves good performance.

## I. Introduction

Hybrid beamforming is an essential part of millimeter wave (mm-Wave) MIMO systems, which can significantly reduce the number of radio frequency (RF) chains in comparison to the number of antennas to achieve the performance close to the fully digital precoding [1]. Since mm-Wave signals are sensitive to blockage and pose a great challenge to establish a reliable non-line-of-sight (NLOS) communication without deploying a relay node or any other suitable component such as intelligent reflecting surface (IRS). Therefore, relay-based hybrid precoding is of great practical importance to enhance network coverage, transmission range and spectral efficiency. To the best of the authors knowledge, hybrid beamforming for single-user multi-relay MIMO communication network, under frequency-selective channels, has not been reported in the literature. It gives motivation for designing the underlying network to significantly improve overall performance [2].

## II. System Model

Considered a single-user multi-relay MIMO system, where multiple antennas are deployed at each communicating unit. The number of antennas at the source, each relay node and destination are denoted as  $N_t$ ,  $N_{rel}$  and  $N_r$ , and the number of RF chains is specified by  $N_t^{RF}$ ,  $N_{rel}^{RF}$  and  $N_r^{RF}$ , respectively. Let  $\mathbf{s} = \{s_1, s_2, \dots, s_{N_s}\} \in \mathbb{C}^{N_s \times 1}$  be a vector consisting of  $N_s$  complex information symbols such that  $\mathbb{E}\{\mathbf{s}_k \mathbf{s}_k^H\} = \mathbf{I}_{N_s}$ . The source transmitted signal at the  $n^{th}$  frequency carrier can be written as

$$\mathbf{x}_S[n] = \mathbf{F}_{RF} \mathbf{F}_{BB}[n] \mathbf{s}[n], \quad (1)$$

where  $\mathbf{F}_{RF} \in \mathbb{C}^{N_t \times N_t^{RF}}$  is the common analog RF precoder and  $\mathbf{F}_{BB}[n] \in \mathbb{C}^{N_t^{RF} \times N_s}$  is the digital baseband precoder corresponding to the  $n^{th}$  frequency channel, respectively. The received signal at the  $i^{th}$  relay node is given as

$$\mathbf{y}_{R,i}[n] = \mathbf{H}_{S-R,i}[n] \mathbf{F}_{RF} \mathbf{F}_{BB}[n] \mathbf{s}[n] + \mathbf{z}_{R,i}[n], i = 1, 2, \dots, N, \quad (2)$$

where  $\mathbf{H}_{S-R,i}[n] \in \mathbb{C}^{N_{rel} \times N_t}$  and  $\mathbf{z}_{R,i}[n] \in \mathbb{C}^{N_{rel} \times 1}$  are the channel matrix between the source and the  $i^{th}$  relay node and, zero mean circularly symmetric complex Gaussian (ZMCSCG) noise with variance  $\sigma_{rel}^2$  i.e.,  $\mathbf{z}_{R,i}[n] \sim \mathcal{CN}(\mathbf{0}, \sigma_{rel}^2 \mathbf{I}_{N_{rel}})$ , respectively. It is important to note that there are two analog processing matrices  $\mathbf{F}_{1,i} \in \mathbb{C}^{N_{rel} \times N_{rel}^{RF}}$  and  $\mathbf{F}_{2,i} \in \mathbb{C}^{N_{rel} \times N_{rel}^{RF}}$ , which are referred to as the common analog RF combiner and precoder, while  $\mathbf{G}_{BB1,i}$  and  $\mathbf{G}_{BB2,i}$  are the frequency dependent baseband processing components at the  $i^{th}$  relay node, respectively. The hybrid relay matrix at the  $i^{th}$  relay node can be expressed as

$$\mathbf{F}_{R,i} = \mathbf{F}_{2,i} \mathbf{G}_{BB,i}[n] \mathbf{F}_{1,i}^H \in \mathbb{C}^{N_{rel} \times N_{rel}}, \quad (3)$$

where  $\mathbf{G}_{BB,i}[n] = (\mathbf{G}_{BB2,i})(\mathbf{G}_{BB1,i})^H \in \mathbb{C}^{N_{rel}^{RF} \times N_{rel}^{RF}}$  is the overall digital baseband processing component. The transmitted signal from the  $i^{th}$  relay node after passing through the relay matrix is given as

$$\mathbf{x}_{R,i}[n] = \mathbf{F}_{R,i} (\mathbf{H}_{S-R,i}[n] \mathbf{x}_S[n] + \mathbf{z}_{R,i}[n]), \quad (4)$$

The combined signal received at the destination due to  $N$  relay nodes can be expressed as

$$\mathbf{y}_D[n] = \sum_{i=1}^N \mathbf{H}_{R-D,i}[n] \mathbf{F}_{R,i}[n] \mathbf{H}_{S-R,i}[n] \mathbf{x}_S[n] + \mathbf{H}_{R-D,i}[n] \mathbf{F}_{R,i}[n] \mathbf{z}_{R,i}[n] + \mathbf{z}_D[n], \quad (5)$$

where  $\mathbf{x}_{R,i}[n] \in \mathbb{C}^{N_{rel} \times 1}$ ,  $\mathbf{H}_{R-D,i}[n] \in \mathbb{C}^{N_r \times NN_{rel}}$ , and  $\mathbf{z}_D[n] \in \mathbb{C}^{N_r \times 1}$  are the relay transmitted vector, channel transfer function between the  $i^{th}$  relay node and destination and, ZMCSCG noise with variance  $\sigma_r^2$  i.e.,  $\mathbf{z}_D[n] \sim \mathcal{CN}(\mathbf{0}, \sigma_r^2 \mathbf{I}_{N_r})$ . Using (6), the received signal at the destination after applying hybrid combiner can be modeled as

$$\mathbf{y}_F[n] = \mathbf{W}_{BB}^H[n] \mathbf{W}_{RF}^H \mathbf{y}_D[n], \quad (6)$$

### III. Analog Beamforming Design

Multi-linear SVD is performed on the overall channel  $\mathbf{H}_{S-R} \in \mathbb{C}^{NN_{rel} \times N_t \times N_{sub}}$  between source and all relay nodes for deriving the common RF precoder  $\mathbf{F}_{RF}$ . Similarly, when the same operation is applied on the combined channel  $\mathbf{H}_{R-D} \in \mathbb{C}^{N_r \times NN_{rel} \times N_{sub}}$  from relay nodes to the destination, it is possible to evaluate the common analog RF combiner ( $\mathbf{W}_{RF}$ ) at the destination. Therefore,

$$\mathbf{H}_{S-R} = \mathbf{S} \times_1 \mathbf{A}_{(1)} \times_2 \mathbf{A}_{(2)} \times_3 \mathbf{A}_{(3)}, \quad (7)$$

where  $\mathbf{S} \in \mathbb{C}^{NN_{rel} \times N_t \times N_{sub}}$  is a core tensor and,  $\mathbf{A}_{(1)} \in \mathbb{C}^{NN_{rel} \times NN_{rel}}$ ,  $\mathbf{A}_{(2)} \in \mathbb{C}^{N_t \times N_t}$  and  $\mathbf{A}_{(3)} \in \mathbb{C}^{N_{sub} \times N_{sub}}$  are unitary matrices consisting of orthonormal basis. The analog precoding matrix  $\mathbf{F}_{RF}$  can be obtained as

$$\mathbf{F}_{RF} = \left( \frac{1}{\sqrt{N_t}} \right) e^{j\angle \mathbf{A}_{(2)}(:, 1:N_t^{RF})}. \quad (8)$$

Multi-linear SVD of  $\mathbf{H}_{R-D}$  can be expressed as

$$\mathbf{H}_{R-D} = \mathbf{S} \times_1 \mathbf{B}_{(1)} \times_2 \mathbf{B}_{(2)} \times_3 \mathbf{B}_{(3)}, \quad (9)$$

Therefore, the desired analog RF combiner  $\mathbf{W}_{RF}$  can be evaluated as

$$\mathbf{W}_{RF} = \left( \frac{1}{\sqrt{N_r}} \right) e^{j\angle \mathbf{B}_{(1)}(:, 1:N_r^{RF})}. \quad (10)$$

For deriving the analog processing matrices at individual relay node, there is a need to apply multi-linear SVD on each pair  $(\mathbf{H}_{S-R,i}[n], \mathbf{H}_{R-D,i}[n])$  of channels by considering all the available sub-carriers i.e.,  $k = 1, 2, \dots, N_{sub}$ . As

$$\mathbf{H}_{SR,i} = \{\mathbf{H}_{S-R,i}[1], \mathbf{H}_{S-R,i}[2], \dots, \mathbf{H}_{S-R,i}[N_{sub}]\}, \quad (11)$$

$$\mathbf{H}_{RD,i} = \{(\mathbf{H}_{R-D,i}[1])^T, (\mathbf{H}_{R-D,i}[2])^T, \dots, (\mathbf{H}_{R-D,i}[N_{sub}])^T\}, i = 1, 2, \dots, N. \quad (12)$$

The required RF beamforming matrices  $\mathbf{F}_{1,i}$  and  $\mathbf{F}_{2,i}$  can be obtained as follows

$$\mathbf{H}_{SR,i} \mathbf{H}_{SR,i}^H = \mathbf{U}_1 \mathbf{S}_1 \mathbf{V}_1^H, \quad \mathbf{H}_{RD,i} \mathbf{H}_{RD,i}^H = \mathbf{U}_2 \mathbf{S}_2 \mathbf{V}_2^H, \quad (13)$$

$$\mathbf{F}_{1,i} = \left( \frac{1}{\sqrt{N_{rel}}} \right) e^{j\angle \mathbf{U}_1(:, 1:N_r^{RF})}, \quad \mathbf{F}_{2,i} = \left( \frac{1}{\sqrt{N_{rel}}} \right) e^{j\angle \mathbf{U}_2(:, 1:N_r^{RF})}, \quad (14)$$

### IV. Digital Precoding and Combining

To complete the hybrid processing design, it is required to obtain the digital baseband processing component at source, destination and relay nodes corresponding to each frequency carrier. As the analog RF beamforming solution at communicating nodes have already been derived therefore, SVD on the equivalent baseband channels lead to the derivation of digital baseband processing components at different communicating nodes. This approach indirectly minimizes the interference among transmitted data streams.

### IV. Computer Simulations

In this section, computer simulations are conducted to evaluate the performance of the proposed scheme. To generate simulation results, sparse mm-Wave channel model [2] in frequency domain is considered, with uniform planar array (UPA), to capture the characteristics of propagation environment.

Fig. 1 shows the spectral efficiency of the proposed approach by changing the number of transmitted data streams i.e.,  $N_s = \{4, 6, 8\}$ , while the number of antennas

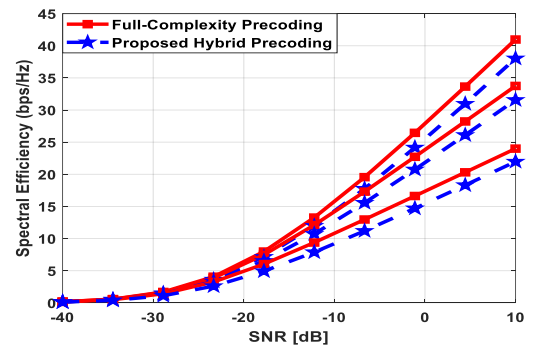


Fig. 1. Spectral Efficiency vs. SNR when  $N_s = \{4, 6, 8\}$  and  $N_t = N_{rel} = N_r = 36$

deployed at each transmitting node is set as  $N_t = N_{rel} = N_r = 36$ . It is clear from the obtained results that the presented hybrid beamforming technique achieves performance close to the full-complexity precoding in a consistent manner.

## ACKNOWLEDGMENT

This work was supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MSIT) (No. NRF-2021R1A2C2005777)

This work was supported by Institute for Information & communications Technology Promotion(IITP) grant funded by the Korea government(MSIT) (No.2017-0-00217, Development of Immersive Signage Based on Variable Transparency and Multiple Layers).

This research was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education(2020R1A6A1A03038540)

\* Corresponding author: Hyoungh-Kyu Song

## REFERENCES

- [1] X. Xue, Y. Wang, L. Dai and C. Masouros, "Relay Hybrid Precoding Design in Millimeter-Wave Massive MIMO Systems," in IEEE Transactions on Signal Processing, vol. 66, no. 8, pp. 2011-2026, 15 April 2018.
- [2] X. Yu, J. Shen, J. Zhang and K. B. Letaief, "Alternating Minimization Algorithms for Hybrid Precoding in Millimeter Wave MIMO Systems," in IEEE Journal of Selected Topics in Signal Processing, vol. 10, no. 3, pp. 485-500, April 2016.